Pearson Edexcel

Mark Scheme (Results)

October 2024

Pearson Edexcel International Advanced Level
In Pure Mathematics (WMA14) Paper 01

Question Number	Scheme	Marks
1(a)	$(8-3x)^{-\frac{1}{3}}=\frac{1}{2}\left(1-\frac{3}{8}x\right)^{-\frac{1}{3}}$	B1
	$\left(1-\frac{3}{8}x\right)^{-\frac{1}{3}}=1+\left(-\tfrac{1}{3}\right)\left(-\frac{3}{8}x\right)+\frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)}{2!}\left(-\frac{3}{8}x\right)^2+\frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{3!}\left(-\frac{3}{8}x\right)^3+\ldots$	M1
	$(8-3x)^{-\frac{1}{3}}=\frac{1}{2}+\frac{1}{16}x+\frac{1}{64}x^2+\frac{7}{1536}x^3+\ldots$	A1 A1
		(4)
(b)	$\frac{1}{2}+\frac{1}{16}\left(\frac{2}{3}\right)+\frac{1}{64}\left(\frac{2}{3}\right)^2+\frac{7}{1536}\left(\frac{2}{3}\right)^3+\ldots=\frac{2851}{5184}\Rightarrow\sqrt[3]{6}=\left(\frac{2851}{5184}\right)^{-1}=\ldots$	M1
	$=\frac{5184}{2851}$ or $1\frac{2333}{2851}$	A1
		(2)
		Total 6

Note a misread of $(8-3x)^{\frac{1}{3}}$ can score a maximum of B1M1A0A0 M0A0

(a)

B1: Obtains $\frac{1}{2}(1\pm\ldots x)^{-\frac{1}{3}}$. $8^{-\frac{1}{3}}$ must be evaluated. May be implied by further work e.g. $\frac{1}{2}\pm\ldots$ provided this has not come from an incorrect method.

e.g. $(8-3x)^{-\frac{1}{3}}=\frac{1}{(8-3x)^{\frac{1}{3}}}=\frac{1}{2}+\frac{1}{\frac{1}{2}\sqrt[3]{3x}}+\frac{1}{\frac{1}{2}\left(\sqrt[3]{3x}\right)^2}+\ldots$ is B0.

M1: Attempts the binomial expansion of $(1 + kx)^n$ to get the **third** or **fourth** term **unsimplified** with an acceptable structure.

The correct binomial coefficient must be combined with x^2 or x^3 which may be unsimplified.

Look for $\frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)}{2!}\ldots x^2$ or $\frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{3!}\ldots x^3$ o.e. (you do not need to be concerned with their $-\frac{3}{8}$ which may be 1 or e.g. if they have a negative sign in front of the whole term)

Do not allow notation such as $\begin{pmatrix}-\frac{1}{3}\\1\end{pmatrix}$, $\begin{pmatrix}-\frac{1}{3}\\2\end{pmatrix}$ unless interpreted correctly in further work.

May be implied by correct coefficients with x^2 or x^3

A1: For 2 correct **simplified** terms of $\frac{1}{16}x$, $\frac{1}{64}x^2$, $\frac{7}{1536}x^3$ (allow x^1 for x for this mark) which may be listed.

Condone if coefficients are given as decimals i.e. $0.0625x$, $0.015625x^2$, $0.004557\ldots x^3$

Also allow for $\left(1-\frac{3}{8}x\right)^{-\frac{1}{3}}=1+\frac{1}{8}x+\frac{1}{32}x^2+\frac{7}{768}x^3+\ldots$ (Note B0M1A1A0 is possible)

A1: $\frac{1}{2}+\frac{1}{16}x+\frac{1}{64}x^2+\frac{7}{1536}x^3$. Isw following a correct answer.

Condone the position of x^n provided it is not clearly on the denominator. e.g. condone $\frac{1}{16}\ x$ but not $\frac{1}{16x}$. Ignore terms with higher powers of *x*. Do not accept $\frac{1}{16}x^1$

(b)

M1: Attempts to substitute $x=\frac{2}{3}$ into their expansion from part (a) to achieve a fraction and attempts to find the reciprocal of that fraction. May be implied by their fraction. You may need to check this on your calculator. They may attempt to find an additional term which is fine. Do not be concerned as to whether the fraction is correct for their binomial expansion if they have shown the substitution. Condone slips including losing one of their terms.

$"\frac{1}{2}+\frac{1}{16}\left(\frac{2}{3}\right)+\frac{1}{64}\left(\frac{2}{3}\right)^2+\frac{7}{1536}\left(\frac{2}{3}\right)^3"=\frac{A}{B}\Rightarrow\frac{B}{A}$ scores M1

$1\div\frac{2851}{5184}$ scores M1

$\frac{1}{"\frac{1}{2}+\frac{1}{16}\left(\frac{2}{3}\right)+\frac{1}{64}\left(\frac{2}{3}\right)^2+\frac{7}{1536}\left(\frac{2}{3}\right)^3"}$ on its own scores M0

A1: $\frac{5184}{2851}$ or $1\frac{2333}{2851}$. Isw once a correct answer is seen. Correct answer scores M1A1.

Question Number	Scheme	Marks
2	Assume that C_1 and C_2 do intersect so that $x^4+10x^2+8=2x^2-7$ (and has real solutions).	B1
	$x^4+8x^2+15=0 \Rightarrow \left(x^2+5\right)\left(x^2+3\right)=0 \Rightarrow x^2=...$ or $x^4+8x^2+15=0 \Rightarrow x^2=\dfrac{-8\pm\sqrt{8^2-4\times15}}{2}$ or $x^4+8x^2+15=0 \Rightarrow \left(x^2+4\right)^2-1=0 \Rightarrow x^2=...$	M1
	$x^2=-5,\ -3$	A1
	e.g. The two values of $x^2<0$ (so $x^4+8x^2+15=0$ has no real roots) which is not possible so C_1 and C_2 do not intersect	A1
		(4)
		Total 4

Note that B0M1A1A1 is possible

B1: Sets up the contradiction by

- making a statement assuming that the two curves intersect e.g. let/assume/suppose C_1 and C_2 / the curves intersect/meet/cross
- equating the curves i.e. $x^4+10x^2+8=2x^2-7$ o.e. (the equation must be correct)

Condone no reference to the equation having real solutions.

M1: Attempts to solve a three-term quadratic equation in x^2 by any valid (**non-calculator**) method. Usual rules apply. **If the roots are just stated then M0A0A0 follows.**

A1: $x^2=-5,\ -3$. Allow e.g. $m^2=-5,\ -3$
Do not condone e.g. $y=-5,\ -3$ without reverting back to $x^2=...$ or making valid deductions about e.g. x^2 being negative

A1: **This mark can only be scored provided the previous M1A1 has been scored.**
Acceptable conclusion which

- refers to x^2 being negative which is not possible or the equation has no real roots or $x^2=-3,-5$ but $x^2 \ldots 0$
- refers to C_1 and C_2 not intersecting
- Correct use of inequalities throughout e.g. $x^2 \ldots 0$ (not just $x^2 \geqslant 0$)

Alternative considering the sign of each term / the expression

B1: As above in main scheme notes

M1: Either one of:

- considers the sign of one side of an equation or inequality which includes either an x^2 or x^4 term. e.g. $x^4 + 8x^2 \geqslant 0$ e.g. $x^4 + 8x^2 + 15 > 0$ Allow to compare with 0 e.g. $x^2 + 3 \neq 0$
- considers the sign of x^2 or x^4 e.g. $x^2 \geqslant 0$
- considers separately at least one part of the product of two terms. e.g. $x^2(x^2 + 8) \geqslant 0$ or $(x^2 + 8) > 0$
- considers the range of values using their completed square form e.g. $\left(x^2 + 4\right)^2 \geqslant 16$ or $x^2 + 4 \geqslant 4$

Do not be concerned by > or $\geqslant$

A1: Correct three-term quadratic in x^2 and correct deductions that e.g. $x^4 \geqslant 0$ and $x^2 \geqslant 0$ depending on their expression (condone > 0)
This mark cannot be scored if the expression they use to make their deductions is incorrect.
e.g. $x^2(x^2 + 8)$ is positive (or zero) since x^2 and $\left(x^2 + 8\right)$ are **both** greater than (or equal to) zero is A1
e.g. $x^4 + 8x^2 \geqslant 0$ since x^2 and x^4 are both $\geqslant 0$ is A1
e.g. $x^2 + 4 \geqslant 4$ since $x^2 \geqslant 0$ is A1
e.g. $x^4 + 8x^2 \geqslant 0$ is A0

A1: **This mark can only be scored provided the previous M1A1 has been scored.**
Acceptable conclusion which refers to

- the equation $x^4 + 8x^2 + 15 = 0$ gives no solutions o.e. e.g. $x^4 + 8x^2 + 15$ cannot equal zero, e.g. $x^4 + 8x^2$ cannot equal -15 e.g. $x^2 + 4$ cannot equal -1
- C_1 and C_2 not intersecting
- Correct use of inequalities throughout e.g. $x^2 \geqslant 0$ (not just $x^2 \geqslant 0$).
  Do not accept e.g. $-3 \leqslant 0$

Question Number	Scheme	Marks
3(a)	$x=3\sin^3\theta \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}\theta}=9\sin^2\theta\cos\theta$ o.e.	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-2\sin 2\theta}{9\sin^2\theta\cos\theta}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-4\sin\theta\cos\theta}{9\sin^2\theta\cos\theta}=-\frac{4}{9}\operatorname{cosec}\theta$	A1
		(3)
(b)	$\theta=\frac{\pi}{6}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\text{"}-\frac{8}{9}\text{"},\ x=3\left(\frac{1}{2}\right)^3=...,\ y=1+\frac{1}{2}=...$	M1
	$y-\frac{3}{2}=-\frac{8}{9}\left(x-\frac{3}{8}\right)$	dM1
	$16x+18y-33=0$	A1
		(3)
(c)	e.g. $x=3\sin^3\theta\Rightarrow\sin\theta=\sqrt[3]{\frac{x}{3}}\Rightarrow\ y=1+\cos 2\theta=1+1-2\sin^2\theta=2-2\left(\frac{x}{3}\right)^{\frac{2}{3}}$	M1
	$y=2-2\left(\frac{x}{3}\right)^{\frac{2}{3}}\Rightarrow 2\left(\frac{x}{3}\right)^{\frac{2}{3}}=2-y\Rightarrow 8x^2=9(2-y)^3$ *	A1*
	$q=3$	B1
		(3)
Alt(c)	e.g. $8x^2=72\sin^6\theta,\quad 9(2-y)^3=9(2-\cos 2\theta-1)^3=9\left(1-\left(1-2\sin^2\theta\right)\right)^3$	M1
	$72\sin^6\theta=9\times8\sin^6\theta\Rightarrow 8x^2=9(2-y)^3$ *	A1*
	$q=3$	B1
		Total 9

(a)

B1: Correct expression for $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ which may be part of their expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. Condone poor labelling/notation provided the intention is clear. Must be in terms of θ

May use identities before differentiating using the product rule e.g. $\sin\theta\left(\frac{1-\cos 2\theta}{2}\right)$

M1: Correct method for $\frac{\mathrm{d}y}{\mathrm{d}x}$. They must be attempting $\frac{\mathrm{d}y}{\mathrm{d}\theta}\times\frac{\mathrm{d}\theta}{\mathrm{d}x}$ o.e. condoning copying slips and condoning poor differentiation, labelling /notation provided the intention is clear.

A1: Achieves $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{4}{9}\operatorname{cosec}\theta$ with no errors but condone invisible brackets and poor notation being recovered. Allow equivalent fractions to $-\frac{4}{9}$. Note $\frac{\mathrm{d}y}{\mathrm{d}x}=$ may be seen in an earlier line of their solution which is acceptable.

Alt (a) Implicit differentiation – note that some candidates may find the cartesian equation and differentiate implicitly or explicitly.

B1: A correctly differentiated cartesian equation. (Do not be concerned with where the cartesian equation has come from for this mark)

e.g. $16x = 9\times3\times(-1)\times(2-y)^2\times\frac{dy}{dx}$

M1: Substitutes in for x and y into their differentiated equation and rearranges to achieve an expression for $\frac{dy}{dx}$ in terms of θ. Condone poor differentiation.

A1: Achieves $\frac{dy}{dx} = -\frac{4}{9}\text{cosec}\,\theta$ with no errors but condone invisible brackets and poor notation being recovered. Allow equivalent fractions to $-\frac{4}{9}$. They must have shown correctly how they found the cartesian equation. i.e. they cannot just use the given result in (c).

(b)

M1: Attempts to find $\frac{dy}{dx}$, x and y when $\theta = \frac{\pi}{6}$. Must achieve values for all three which may be implied by further work. Condone slips but if values are only stated then their $\frac{dy}{dx}$ must be correct for their k (look for $2\times$ their k) and one of x or y must be correct.

Note that they may use their $\frac{dy}{dx}$ in terms of x and y $\left(= "-\frac{16x}{27(2-y)^2}"\right)$ by either finding the differentiated equation in (a) or by converting in (b).

dM1: Complete method for the tangent using their values. It is dependent on the previous method mark. Condone one sign error when substituting in the coordinates. If they use $y = mx + c$ they must proceed as far as $c = ...$

A1: $16x + 18y - 33 = 0$ or any integer multiple of this equation. Must have = 0.

(c)

M1: A complete method to eliminate θ. Any identities used must be correct when substituted in. Do not allow the parametric equations to be expressed as θ = and equated. May also be awarded if seen in (a).

A1*: Correct proof with no errors including brackets although condone missing trailing brackets if it does not affect the expression. Must be seen in (c).

B1: Correct value of q. Do not accept $x = 3$ Can be scored for the domain $-3 \leqslant x \leqslant 3$ which must be in terms of x

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Alt(c)

M1: A complete method to express lhs and rhs both in terms of sin θ only or both in terms cos 2θ only. Any identities used must be correct when substituted in. They cannot proceed from $9(2-y)^3 = 72\sin^6\theta$ without an intermediate stage but condone $8x^2 = 72\sin^6\theta$

A1*: Fully correct work showing lhs = rhs and gives a conclusion. There should be no errors including brackets but condone missing trailing brackets if it does not affect the expression.

B1: As above in the notes for (c)

Question Number	Scheme	Marks
4(a)	$x=1, y=2 \Rightarrow 3x^2+2y^2-4xy+8^x-11=3+8-8+8-11=0*$	B1*
		(1)
(b)	$8^x \to 8^x \ln 8$	B1
	$4xy \to 4x\dfrac{dy}{dx}+4y$ or $y^2 \to 2y\dfrac{dy}{dx}$	M1
	$6x+4y\dfrac{dy}{dx}-4y-4x\dfrac{dy}{dx}+8^x\ln 8=0$ o.e.	A1
	$\dfrac{dy}{dx}(4y-4x)=4y-6x-8^x\ln 8 \Rightarrow \dfrac{dy}{dx}=...$	M1
	$\dfrac{dy}{dx}=\dfrac{4y-6x-8^x\ln 8}{4y-4x}$ o.e. e.g $\dfrac{dy}{dx}=\dfrac{6x+8^x\ln 8-4y}{4x-4y}$ or e.g $\dfrac{dy}{dx}=\dfrac{4y-6x-e^{x\ln 8}\ln 8}{4y-4x}$	A1
		(5)
(c)	$(1, 2) \to \dfrac{dy}{dx}=\dfrac{8-6-8\ln 8}{8-4} \Rightarrow y-2=\dfrac{4}{8\ln 8-2}(x-1)$	M1
	$y=0 \Rightarrow 0-2=\dfrac{4}{8\ln 8-2}(x-1) \Rightarrow x=...$	M1
	$x=2-12\ln 2$	A1
		(3)
		Total 9

(a)

B1*: Substitutes x = 1 and y = 2 and obtains 0 with no errors.
Minimum required without evaluating each term e.g. $3\times1+2\times2^2-4\times1\times2+8^1-11=0$. We just need a correct equation which evaluates as 0.
Alternatively, substitutes in x = 1, forms a quadratic in y i.e. $2y^2-4y=0$ o.e. and solves to find y via any method:
$3+2y^2-4y+8-11=0 \Rightarrow 2y^2-4y=0 \Rightarrow y=2$ (ignore the other solution 0)
They may at this stage substitute in y = 2 to get 0.
In any of the approaches a conclusion is not required.

(b)

B1: Correct differentiation of 8^x. Award for sight of $\ln 8\times 8^x$ or $e^{x\ln 8}\times\ln 8$. Do not be concerned with the use of modulus signs around ln8

M1: For $y^2 \to ky\dfrac{dy}{dx}$ or $4xy \to Ax\dfrac{dy}{dx}+By$

A1: Fully correct differentiation. Condone the omission of = 0 or may be implied by further work. Accept $6x+4y\dfrac{dy}{dx}-4y-4x\dfrac{dy}{dx}+e^{x\ln 8}\ln 8=0$
Ignore if they start their differentiated line with e.g. $\dfrac{dy}{dx}=$

M1: Attempts to make $\frac{dy}{dx}$ the subject with **2 terms** in $\frac{dy}{dx}$ coming from the correct places. i.e. they must have two terms (one of each) of the form $\alpha x\frac{dy}{dx}$ and $\beta y\frac{dy}{dx}$ where α, β are non-zero constants.

A1: Any correct expression for $\frac{dy}{dx}$. Allow $y' = ...$ o.e. isw after a correct answer is seen

Condone poor dividing lines which do not go quite far enough provided an incorrect method was not seen. E.g. $\frac{4y-6x-8^x \ln 8}{4y-4x}$

(c) **Note that an incorrect derivative in part (b) can score maximum 110 in part (c)**

M1: Attempts to use $x = 1$ and $y = 2$ in their $\frac{dy}{dx}$ to find the gradient at P (condone if this is a decimal) and attempts to form the equation of the normal at P using the negative reciprocal gradient. Condone one sign slip when substituting in the coordinates into the straight line equation.

They may set $y = 0$ at this stage such that $-2 = \frac{4}{8\ln 8 - 2}(x-1)$ which is acceptable.

If they use $y = mx + c$ they must proceed as far as $c = ...$

M1: States $y = 0$ or substitutes $y = 0$ into their straight-line equation using a changed gradient from their $\frac{dy}{dx}$ and rearranges to find x. Do not be concerned by the mechanics of their rearrangement.

A1: Correct expression in the required form

Question Number	Scheme	Marks
5(a)	e.g. $\frac{r}{h}=\frac{12}{30}\Rightarrow r=\frac{12}{30}h\Rightarrow V=\frac{1}{3}\pi r^2h=\frac{1}{3}\pi\left(\frac{2}{5}h\right)^2 h$	M1
	$V=\frac{4\pi h^3}{75}$ *	A1*
		(2)
(b)	$V=\frac{4\pi h^3}{75}\Rightarrow\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{12\pi h^2}{75}\quad\left(=\frac{4\pi h^2}{25}\right)$	B1
	$V=1.5\times60\times2\pi$ $180\pi=\frac{4\pi h^3}{75}\Rightarrow h^3=3375\Rightarrow h=...(15)$	M1
	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{75}{12\pi\times15^2}\times2\pi=...$	M1
	$\frac{1}{18}$ (cm s^{-1})	A1
		(4)
		Total 6

(a)

M1: Uses the 12 and the 30 to find the ratio between *h* and *r* and substitutes into the volume formula to find *V* in terms of *h*. Condone poor bracketing / invisible brackets for this mark.

May just state $r=\frac{2}{5}h$

This mark cannot be for substituting in e.g. $r=\frac{2}{5}$ instead of $r=\frac{2}{5}h$

A1*: Correct proof with no errors including brackets. There must be at least one intermediate stage of working between establishing $r=\frac{12}{30}h$ and proceeding to the given answer.

(b) **Note that substituting $h=1.5$ or $h=90$ will score M0M0 in (b)**

B1: Correct expression for $\frac{\mathrm{d}V}{\mathrm{d}h}$ in any form

M1: Attempts their volume ("$2\pi\times t$") after 1.5 **minutes** and uses this with the given result from (a) to find *h*. They may find *h* by equating their expression for the volume to the given result from (a) before substituting $t=90$ and rearranging to find *h*. Do not be concerned by the mechanics of the rearrangement. Condone slips.

M1: Correct chain rule attempt using their *h* to find $\frac{\mathrm{d}h}{\mathrm{d}t}$. It is dependent on having used $t = 90$ (or condone $t = 1.5$) when finding *h*.

A1: Correct value. Allow awrt 0.0556 (cm s^{-1}) Units not required for this mark.

Alt(b) I Solving the differential equation $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{75}{6h^2}$ and then using $t=90$

B1: $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{12\pi h^2}{75}\left(=\frac{4\pi h^2}{25}\right)$

M1: Attempts $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}\left(=\frac{75}{6h^2}\right)$ to achieve an expression in *h*, attempts to solve their differential equation $\int 6h^2\,\mathrm{d}h=\int 75\,dt \Rightarrow 2h^3=75t+c$ o.e. and uses $t=0,\ h=0\Rightarrow c=0$ (you may not see the working for *c*) **to find *h* = f(*t*)** ($h=(37.5t)^{\frac{1}{3}}$). Do not be concerned by the mechanics of the rearrangement.

M1: Attempts $\frac{\mathrm{d}h}{\mathrm{d}t}\left(=\frac{1}{3}\sqrt[3]{37.5}\,t^{-\frac{2}{3}}\right)$ and substitutes $t=90\Rightarrow\frac{\mathrm{d}h}{\mathrm{d}t}=\ldots$

A1: As above in main scheme

Alt(b) II Equating volumes, rearranging to $h=\ldots$ and then using $t=90$

B1: $V=2\pi t$

M1: Equates their $2\pi t$ equal to $\frac{4\pi h^3}{75}$ and rearranges to $h=\mathrm{f}(t)$ ($h=(37.5t)^{\frac{1}{3}}$). Do not be concerned by the mechanics of the rearrangement.

M1: Attempts $\frac{\mathrm{d}h}{\mathrm{d}t}\left(=\frac{1}{3}\sqrt[3]{37.5}\,t^{-\frac{2}{3}}\right)$ and substitutes $t=90\Rightarrow\frac{\mathrm{d}h}{\mathrm{d}t}=\ldots$

A1: As above in main scheme

Question Number	Scheme	Marks
6	$u=\sqrt{x^3+1}\Rightarrow u^2=x^3+1\Rightarrow 2u\dfrac{\mathrm{d}u}{\mathrm{d}x}=3x^2$ or $u=\sqrt{x^3+1}\Rightarrow u^2=x^3+1\Rightarrow 2u=3x^2\dfrac{\mathrm{d}x}{\mathrm{d}u}$ or $u=\sqrt{x^3+1}\Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x}=\dfrac{1}{2}\left(x^3+1\right)^{-\frac{1}{2}}\times 3x^2$ or $x=(u^2-1)^{\frac{1}{3}}\Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}u}=\dfrac{2}{3}u(u^2-1)^{-\frac{2}{3}}$	B1
	e.g. $\displaystyle\int\frac{9x^5}{\sqrt{x^3+1}}\mathrm{d}x=\int\frac{9x^5}{u}\frac{2u}{3x^2}\mathrm{d}u=\int 6x^3\,\mathrm{d}u=6\int\left(u^2-1\right)\mathrm{d}u$ or e.g. $\displaystyle\int\frac{9x^5}{\sqrt{x^3+1}}\mathrm{d}x=\int\frac{9x^5}{\sqrt{x^3+1}}\frac{2\sqrt{x^3+1}}{3x^2}\mathrm{d}u=\int 6x^3\,\mathrm{d}u=6\int\left(u^2-1\right)\mathrm{d}u$	M1A1
	$\displaystyle 6\int\left(u^2-1\right)\mathrm{d}u=2u^3-6u\ (+c)=2\left(x^3+1\right)^{\frac{3}{2}}-6\left(x^3+1\right)^{\frac{1}{2}}\ (+c)$	M1
	$=2\left(x^3+1\right)^{\frac{1}{2}}\left[\left(x^3+1\right)-3\right]+c=2\left(x^3+1\right)^{\frac{1}{2}}\left(x^3-2\right)+c$	A1
		(5)
		Total 5

B1: Any correct equation involving $\dfrac{\mathrm{d}u}{\mathrm{d}x}$ or $\dfrac{\mathrm{d}x}{\mathrm{d}u}$ or du and dx separately. May be implied by further work.

M1: Attempts to fully change $\displaystyle\int\frac{9x^5}{\sqrt{x^3+1}}\mathrm{d}x$ into an integral with respect to u. There must be no x's anywhere. Must include an attempt at replacing dx to get du so M0 if there is no du term present (unless implied by further work) or dx becomes du without an attempt at connecting them first. (i.e. there must have been an attempt at $\dfrac{\mathrm{d}u}{\mathrm{d}x}$ or $\dfrac{\mathrm{d}x}{\mathrm{d}u}$). Condone no integral sign for this mark.

A1: Reaches $\displaystyle 6\int\left(u^2-1\right)\mathrm{d}u$ o.e. Condone the omission of the integral sign but they must have a du or implied by further work.

M1: Attempts to integrate their expression in terms of u only (usually $\int(Au^2+B)\,\mathrm{d}u$) **and** attempts to reverse the substitution so that the expression is completely in terms of x (with or without the constant of integration). For the integration look for a correct method to integrate a term i.e. $...u^n\to...u^{n+1}$ or $B\to Bu$ or $\dfrac{C}{u}\to\ln u$ at least once. Just seeing + c is not an example of integrating.

A1: Correct expression in the required form including the + c

Question Number	Scheme	Marks
7(a)	$\dfrac{3x-1}{x+2}=3+\dfrac{\dots}{x+2}$ or $\dots-\dfrac{7}{x+2}$	M1
	$\dfrac{3x-1}{x+2}=3-\dfrac{7}{x+2}$	A1
		(2)
(b)	$V=(\pi)\int\left(3-\dfrac{7}{x+2}\right)^2\mathrm{d}x=\dots$	M1
	$=(\pi)\int\left(9-\dfrac{42}{x+2}+\dfrac{49}{(x+2)^2}\right)\mathrm{d}x=\dots-42\ln(x+2)+\dots$ **or** $=(\pi)\int\left(9-\dfrac{42}{x+2}+\dfrac{49}{(x+2)^2}\right)\mathrm{d}x=\dots+\dots-\dfrac{49}{x+2}$	M1
	$=(\pi)\int\left(9-\dfrac{42}{x+2}+\dfrac{49}{(x+2)^2}\right)\mathrm{d}x=\dots-42\ln(x+2)+\dots$ **and** $=(\pi)\int\left(9-\dfrac{42}{x+2}+\dfrac{49}{(x+2)^2}\right)\mathrm{d}x=\dots+\dots-\dfrac{49}{x+2}$	dM1
	$=(\pi)\int\left(9-\dfrac{42}{x+2}+\dfrac{49}{(x+2)^2}\right)\mathrm{d}x=9x-42\ln(x+2)-\dfrac{49}{x+2}$	A1
	$V=(\pi)\left[9x-42\ln(x+2)-\dfrac{49}{x+2}\right]_1^4=(\pi)\left\{36-42\ln 6-\dfrac{49}{6}-\left(9-42\ln 3-\dfrac{49}{3}\right)\right\}$	M1
	$=\pi\left(\dfrac{211}{6}-42\ln 2\right)$	A1
		(6)
		Total 8

(a)

M1: Obtains either $A = 3$ or $B=-7$

A1: $3-\dfrac{7}{x+2}$ Just stating the values of A and B is insufficient but you may award this mark if it is seen in (b). Allow $3+\dfrac{-7}{x+2}$ and $3+-\dfrac{7}{x+2}$

(b) **Condone transcription errors for the method marks** e.g. $\dfrac{\dots}{x+2}\to\dots\ln(x-2)$ **provided no incorrect method is seen. Condone invisible brackets for the M marks**

M1: Uses their result from part (a) and applies the volume formula (with or without the π and condone 2π) and attempts to square the bracket. Condone if they just square the two terms in the brackets or if their squaring is poor.

M1: Integrates to obtain the correct form for one of the fractional terms i.e. $\frac{...}{x+2} \to ...\ln(x+2)$ or $\frac{...}{(x+2)^2} \to \frac{...}{x+2}$. This can be scored even if they have not squared any terms originally if they have one of these terms of the required form. Condone invisible brackets for $\ln x+2$

dM1: Integrates to obtain the correct form for both fractional terms i.e. $\frac{...}{x+2} \to ...\ln(x+2)$ and $\frac{...}{(x+2)^2} \to \frac{...}{x+2}$. It is dependent on the previous method mark. Condone invisible brackets for $\ln x+2$

A1: $9x-42\ln(x+2)-\frac{49}{x+2}$ o.e. May be implied by further work.

M1: Applies the limits 1 and 4 to an expression of the form $Ax \pm B\ln(x+2) \pm C(x+2)^n$ where $A, B, C, n \neq 0$ and subtracts the correct way round. The expression with values embedded is sufficient. Condone slips and invisible brackets.

A1: $\pi\left(\frac{211}{6}-42\ln 2\right)$

Correct expression in the **required form**. The fraction does not need to be in simplest form.

Alternative methods (including not using part (a)) requiring use of a substitution e.g. $u = x+2$ or e.g. $u = \frac{7}{x+2}$**. Watch out for these longer methods which can still score full marks.**

M1: If using $u = x+2$ then $(\pi)\int\left(\frac{3x-1}{x+2}\right)^2 \mathrm{d}x \Rightarrow (\pi)\int 9-\frac{42}{u}+\frac{49}{u^2}\,\mathrm{d}u$ (They will need to divide their numerator by the denominator to achieve $\alpha+\frac{\beta x+\gamma}{(x+2)^2}$). Condone slips.

If using $u = \frac{7}{x+2}$ then $(\pi)\int \frac{42}{u}-\frac{63}{u^2}-7\,\mathrm{d}u$ Condone slips.

M1: As above in the main scheme notes for one of the fractional terms i.e. $\frac{...}{u} \to ...\ln u$ or $\frac{...}{u^2} \to \frac{...}{u}$.

dM1: Integrates to obtain the correct form for both fractional terms

A1: $9u-42\ln u-\frac{49}{u}$ if using $u = x+2$ **or** $42\ln u+\frac{63}{u}-7u$ if using $u = \frac{7}{x+2}$

M1: Applies the limits 1 and 4 to an expression of the required form (see above in main scheme notes) if the expression is converted back to being in terms of x
or applies the limits 3 and 6 if in terms of u if using $u = x+2$
or e.g. $\frac{7}{3}, \frac{7}{6}$ if using $u = \frac{7}{x+2}$

A1: As above in main scheme notes.

Question Number	Scheme	Marks
8(a)	$\pm\left(-6\mathbf{i}+4\mathbf{j}-\mathbf{k}-\left(-10\mathbf{i}+5\mathbf{j}-4\mathbf{k}\right)\right)$	M1
	e.g. $\mathbf{r}=-10\mathbf{i}+5\mathbf{j}-4\mathbf{k}\pm\lambda\left(4\mathbf{i}-\mathbf{j}+3\mathbf{k}\right)$ e.g. $\mathbf{r}=-6\mathbf{i}+4\mathbf{j}-\mathbf{k}\pm\lambda\left(4\mathbf{i}-\mathbf{j}+3\mathbf{k}\right)$	A1
		(2)
(b)	$3+3\mu=-6\Rightarrow\mu=-3$	M1
	$p+12=4\Rightarrow p=...$ or $q-3=-1\Rightarrow q=...$	dM1
	$p=-8,\quad q=2$	A1
		(3)
(c)	$\left(4\mathbf{i}-\mathbf{j}+3\mathbf{k}\right)\cdot\left(3\mathbf{i}-4\mathbf{j}+\mathbf{k}\right)=12+4+3$	M1
	$\left(4\mathbf{i}-\mathbf{j}+3\mathbf{k}\right)\cdot\left(3\mathbf{i}-4\mathbf{j}+\mathbf{k}\right)=\sqrt{26}\sqrt{26}\cos\theta=19\Rightarrow\cos\theta=...$	M1
	$(\cos\theta=)\ \frac{19}{26}$	A1
		(3)
(d)	$AC=AB\sin\theta=\sqrt{26}\sqrt{1-\left(\frac{19}{26}\right)^2}$	M1
	$\frac{3\sqrt{910}}{26}$ o.e.	A1
		(2)
		Total 10

(a)

M1: Attempts the direction of l_1. The expression is sufficient condoning one sign slip or may be implied by 2 correct components of $\pm\left(4\mathbf{i}-\mathbf{j}+3\mathbf{k}\right)$

A1: Any correct equation including '**r** ='. Allow any parameter to be used including μ.

Allow $\mathbf{r}=-10\mathbf{i}+5\mathbf{j}-4\mathbf{k}\pm\lambda\left(4\mathbf{i}-\mathbf{j}+3\mathbf{k}\right)$, $\mathbf{r}=\begin{pmatrix}-10\\5\\-4\end{pmatrix}\pm\lambda\begin{pmatrix}4\\-1\\3\end{pmatrix}$, $\mathbf{r}=\begin{pmatrix}-6\\4\\-1\end{pmatrix}\pm\lambda\begin{pmatrix}-4\\1\\-3\end{pmatrix}$

Do not accept i, j, k notation in either bracket using column vector notation

e.g. $\mathbf{r}=\begin{pmatrix}-10\\5\\-4\end{pmatrix}\pm\lambda\begin{pmatrix}4\mathbf{i}\\-1\mathbf{j}\\3\mathbf{k}\end{pmatrix}$

(b)

M1: Equates the x component of l_2 to -6 and proceeds to find a value for μ.

dM1: Uses the value of μ to find p or q. It is dependent on the previous method mark.
Alternatively,

- forms a correct equation for the x component using their l_1 and l_2
- proceeds to find λ
- uses this value of λ with the value of μ correctly to find p or q by forming an equation for the y or z component.

Do not be concerned by the mechanics of the rearrangement in solving.

A1: Correct values

Alt (b)

M1: Forms three correct equations $3+3\mu=-6,\ p-4\mu=4,\ q+\mu=-1$

dM1: Solves simultaneously to find a value for *p* or *q*. Do not be concerned by the mechanics of the rearrangement.

A1: Correct values

(c)

M1: Attempts the scalar product using their direction vectors or may find another direction vector which should be a multiple of the one for l_1 or l_2 (condoning slips e.g. if they restart using another point on l_2 to find a direction vector). Look for at least two correct products using their l_1. May be implied by "±19". May find the obtuse angle which can still score this mark.

M1: Completes the scalar product method using their direction vectors to find cos θ (acute or obtuse). Attempts the magnitude of both of their direction vectors and uses these values in the correct positions in the equation for cos θ

A1: $\frac{19}{26}$ or exact equivalent. Do not accept fractions where the numerator or denominator is not an integer (square roots must be evaluated). Isw if they proceed to find θ

Alt (c) There will be many alternatives such as the one below. Send to review if unsure.

M1: The first mark should be for finding the required components to use in the cosine rule e.g. attempts to find another point *P* on l_2 and attempts to find the lengths of *AB*, *BP* and *PA*

M1: The second mark should be using all the components required in the cosine rule. E.g.uses their lengths for *AB*, *BP* and *PA* in the correct positions in the cosine rule.

A1: As above in the main scheme notes.

(d)

M1: Fully correct method to find the exact value for the length of *AC*.
May alternatively attempt e.g.

$$\begin{pmatrix}3\\-4\\1\end{pmatrix}\bullet\begin{pmatrix}13+3\mu\\-13-4\mu\\6+\mu\end{pmatrix}=0\Rightarrow\mu=-\frac{97}{26}\Rightarrow\overrightarrow{AC}=\begin{pmatrix}\frac{47}{26}\\\frac{25}{13}\\\frac{59}{26}\end{pmatrix}\Rightarrow AC=\sqrt{\left(\frac{47}{26}\right)^2+\left(\frac{25}{13}\right)^2+\left(\frac{59}{26}\right)^2}$$

Note the coordinates for *C* are $\left(-\frac{213}{26},\frac{90}{13},-\frac{45}{26}\right)$ and they may use their coordinates for *C* with point *A* to find the distance *AC*. Condone one sign slip in the distance formula. i.e. there must be sufficient evidence that they were attempting the differences between appropriate coordinates.
Look out for other fully correct methods to find the exact value for the length of *AC*.
e.g. attempts to find $|BC|$ and uses either trigonometry or Pythagoras' Theorem.

A1: Correct value. Allow $\sqrt{\frac{315}{26}}$ or $3\sqrt{\frac{35}{26}}$

Question Number	Scheme	Marks
9(a)	$\frac{1}{x(2x-1)}\equiv\frac{A}{x}+\frac{B}{2x-1}\Rightarrow 1\equiv A(2x-1)+Bx\Rightarrow A=\ldots\text{or } B=\ldots$	M1
	$\frac{1}{x(2x-1)}\equiv\frac{2}{2x-1}-\frac{1}{x}$	A1
		(2)
(b)	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{1}{50}h(2h-1)\cos\left(\frac{t}{10}\right)\Rightarrow\int\frac{1}{h(2h-1)}\mathrm{d}h=\int\frac{1}{50}\cos\left(\frac{t}{10}\right)\mathrm{d}t$	M1
	$\int\frac{1}{h(2h-1)}\mathrm{d}h=\int\left(\frac{2}{2h-1}-\frac{1}{h}\right)\mathrm{d}h=\ln(2h-1)-\ln h$	M1
	$\int\left(\frac{2}{2h-1}-\frac{1}{h}\right)\mathrm{d}h=\int\frac{1}{50}\cos\left(\frac{t}{10}\right)\mathrm{d}t\Rightarrow\ln(2h-1)-\ln h=\frac{1}{5}\sin\left(\frac{t}{10}\right)\quad(+c)$	A1ft
	e.g. $t=0,h=2.5\Rightarrow\ln(2\times2.5-1)-\ln 2.5=c$ or e.g. $\ln\frac{2h-1}{h}=\frac{1}{5}\sin\left(\frac{t}{10}\right)+c\Rightarrow\frac{2h-1}{h}=Xe^{\frac{1}{5}\sin\left(\frac{t}{10}\right)}\quad t=0,h=2.5\Rightarrow X=1.6$	M1
	$\ln(2h-1)-\ln h=\frac{1}{5}\sin\left(\frac{t}{10}\right)+\ln 1.6\Rightarrow\frac{2h-1}{h}=1.6e^{\frac{1}{5}\sin\left(\frac{t}{10}\right)}$ $\Rightarrow 2h-1.6he^{\frac{1}{5}\sin\left(\frac{t}{10}\right)}=1\Rightarrow h=\ldots$	M1
	$h=\frac{5}{10-8e^{\frac{1}{5}\sin\left(\frac{t}{10}\right)}}$	A1
		(6)
(c)	$\sin\left(\frac{t}{10}\right)=1\Rightarrow\frac{t}{10}=\frac{\pi}{2},\frac{5\pi}{2},\frac{9\pi}{2},\ldots\Rightarrow t=\ldots$	M1
	$t=45\pi=141$ (s)	A1
		(2)
		Total 10

(a)

M1: Correct attempt at partial fractions. Sets $\frac{1}{x(2x-1)}=\frac{A}{x}+\frac{B}{2x-1}$ and uses a correct method to identify at least one constant. Incorrect work e.g. $\frac{A}{x}+\frac{B}{2x-1}\Rightarrow 1=Ax+B(2x-1)$ etc. is M0.

A1: Correct partial fractions. Note that this mark is not just for the correct constants, it is for the correct written fractions but allow this mark to be scored if it is seen in part (b) (in terms of *h* or *x*).

Condone $\frac{2}{2x-1}+\frac{-1}{x}$ or written as e.g. $2(2x-1)^{-1}-x^{-1}$

(b) **Note that they may work in x throughout, but the final answer must be $h = ...$**

M1: Attempts to separate the variables. Do not be concerned by slips with the constants so look for as a minimum $\int \frac{1}{h(2h-1)} = \int \cos(...t)$ o.e. May omit an integral sign on one of the sides or may be implied by their integrated expression.

M1: For $\int \left(\frac{A}{h} + \frac{B}{2h-1}\right) \mathrm{d}h = \alpha \ln h + \beta \ln(2h-1)$

or e.g. $\lambda \int \left(\frac{"A"}{h} + \frac{"B"}{2h-1}\right) \mathrm{d}h = \alpha \ln h + \beta \ln(2h-1)$ (if they attempt some rearranging first)

Condone invisible brackets.

A1ft: Fully correct equation following through their A and B. The "$+c$" is not required here.

$$\int \left(\frac{A}{h} + \frac{B}{2h-1}\right) \mathrm{d}h = \int \frac{1}{50} \cos\left(\frac{t}{10}\right) \mathrm{d}t \Rightarrow A \ln h + \frac{B}{2} \ln(2h-1) = \frac{1}{5} \sin\left(\frac{t}{10}\right) \ (+c) \text{ o.e.}$$

Condone invisible brackets.

M1: States or uses $t = 0$, $h = 2.5$ followed by $c = ...$ is sufficient for M1. You do not need to check their substitution/rearrangement but if there is just a value for the constant then you may need to check they used $t = 0$, $h = 2.5$. May be implied by e.g. $\ln 8 - \ln 5$, $\ln 1.6$, awrt 0.47 following correct integration. Note that they may have removed lns first.

M1: To score this mark they must have integrated and achieved an expression of the form

$$\pm p \ln h \pm q \ln(2h-1) = \pm r \sin\left(\frac{t}{10}\right) + c \text{ o.e.}$$

Rearranges using correct algebra to make h the subject including when combining logs and when dealing with the constant. May occur before finding their constant but there must have been a constant.

Expect to see terms in h collected on one side as part of their manipulation rather than proceeding to the given answer once they have established the value of k. Alternatively, they may cancel e.g. $\frac{2h-1}{h} \to 2 - \frac{1}{h}$ and rearrange to make h the subject using correct algebra.

Do not condone sign slips for this mark.

A1: Correct expression (including $h = ...$) in the required form. Requires

- Correct integration
- Correct algebra

  Do not penalise brackets recovered in their work or working in x rather than h until the final answer.

(c) **Note that an answer with no working seen scores M0A0**

M1: Solves $\sin\left(\frac{t}{10}\right) = 1$ to find a value for t. Do not allow this mark to be implied by

e.g. 5π, 25π, 45π but allow for e.g. $\frac{t}{10} = \frac{\pi}{2} \Rightarrow t = ..$ Must be working in radians.

Note that attempts that do not use part (b) and just use the differential equation proceeding to $\cos\left(\frac{t}{10}\right) = 0$ score M0A0. (**Note if their k is negative condone if they solve** $\sin\left(\frac{t}{10}\right) = -1$)

A1: awrt 141 (but do not allow just 45π) Units not required.

Question Number	Scheme	Marks
10(a)	$\frac{\mathrm{d}x}{\mathrm{d}t}=6t$ o.e.	B1
	$\text{Area}=\left(\int \sin t\sin 2t\times 6t\,\mathrm{d}t=\right)\int \sin t\times 2\sin t\cos t\times 6t\,\mathrm{d}t$	M1
	$=12\int_0^{\frac{\pi}{2}} t\sin^2 t\cos t\,\mathrm{d}t$	A1
		(3)
(b)	$=(12)\left[\frac{t}{3}\sin^3 t\right]_0^{\frac{\pi}{2}}-(12)\int_0^{\frac{\pi}{2}}\frac{1}{3}\sin^3 t\,\mathrm{d}t$	M1
	$=(12)\left[\frac{t}{3}\sin^3 t\right]_0^{\frac{\pi}{2}}-\left(\frac{12}{3}\right)\int_0^{\frac{\pi}{2}}\sin t\left(1-\cos^2 t\right)\mathrm{d}t$	M1
	$=\left[4t\sin^3 t+4\cos t-\frac{4}{3}\cos^3 t\right]_0^{\frac{\pi}{2}}$	A1
	$=2\pi-\left(4-\frac{4}{3}\right)$	M1
	$=2\pi-\frac{8}{3}$	A1
		(5)
		Total 8

(a) On EPEN this is M1M1A1 but we are marking this as B1M1A1

It is acceptable to work in a different variable such as θ instead of t provided the final answer is in terms of t but withhold the final mark if there is a mixture of variables in the same line of working.

B1: $\frac{\mathrm{d}x}{\mathrm{d}t}=6t$ seen or implied.

$6t\sin t\sin 2t$ implies B1 but $6t\sin^2 t\cos t$ does not imply B1.

M1: Uses $\sin 2t=2\sin t\cos t$ within the expression for y i.e. $y=\sin t\sin 2t=2\sin^2 t\cos t$ or may be seen in their integral. May be implied by e.g. $\int pt\sin t\sin 2t\ (\mathrm{d}t)\rightarrow\int 2pt\sin^2 t\cos t\ (\mathrm{d}t)$

Condone the omission of *t*. i.e. $y=2\sin^2\cos$

A1: Correct integral including $\mathrm{d}t$ with k = 12 **with at least one intermediate stage of working** (which may only be e.g. $\frac{\mathrm{d}x}{\mathrm{d}t}=6t$) The integral sign, d*t*, limits only need to be present in the final answer. Ignore any attempts to verify the limits of 0 and $\frac{\pi}{2}$.

Do not condone poor notation e.g. $\sin t^2$ or the absence of variables in the main body of the solution.

$12t\sin^2 t\cos t$ with or without integration notation and with no other working implies B1M1A0

(b) Condone working in mixed variables provided the intention is clear or they recover in further work. Condone the absence of variables in trigonometric functions e.g. sin for $\sin t$ or cos for $\cos t$

M1: Attempts integration by parts in the correct direction to achieve an expression of the form $...t\sin^3 t \pm ...\int \sin^3 t \text{ d}t$ Condone the omission of their 12

M1: Uses $\sin^3 t = \sin t\left(1-\cos^2 t\right)$ to reach the integrable form $...t\sin^3 t \pm ...\int \sin t(1-\cos^2 t) \text{ d}t$ o.e.

May be seen as $...t\sin^3 t \pm ...\int \sin t - \sin t\cos^2 t \text{ d}t$.

Alternatively uses compound angle formule to substitute $\sin^3 t = ...\sin t \pm ...\sin 3t$ to reach an integrable form. $...t\sin^3 t \pm ...\int\left(...\sin t \pm ...\sin 3t\right)\text{d}t$ $\left(4t\sin^3 t - \int(3\sin t - \sin 3t)\text{d}t\right)$

In either approach it may be implied by their integrated expression. Condone the omission of their 12

A1: Fully correct integration of $\int t\sin^2 t\cos t \text{ d}t$

or alternatively $4t\sin^3 t - \int(3\sin t - \sin 3t)\,\text{d}t = 4t\sin^3 t + 3\cos t - \frac{1}{3}\cos 3t$.

Condone any multiple and condone missing variables e.g. $4t\sin^3 + 4\cos - \frac{4}{3}\cos^3$

M1: Applies the limits 0 and $\frac{\pi}{2}$ to an expression of the form $At\sin^3 t + B\cos t + C\cos^3 t$ where $A, B, C \neq 0$ or in the alternative $\alpha\cos t + \beta\cos 3t$ where $\alpha, \beta \neq 0$

A1: Correct answer in the required form. $\frac{8}{3}$ does not need to be in simplest form. isw once a correct answer is seen.

Alternative substitution method using $u = \sin t$

M1: Attempts to use the substitution $u = \sin t$ to proceed to $(12)\int u^2\sin^{-1}u\,\text{d}u$ and attempts integration by parts in the correct direction to achieve an expression of the form $(12)\left(\frac{u^3}{3}\sin^{-1}u - \frac{1}{3}\int\frac{u^3}{\sqrt{1-u^2}}\text{d}u\right)$ but condone coefficient slips.

Condone the omission of their 12

M1: Proceeds to an integrable form by using the substitution $v = \sqrt{1-u^2}$

e.g. $(12)\left(\frac{1}{3}u^3\sin^{-1}u + \frac{1}{3}\int\left(1-v^2\right)\text{d}v\right)$ Condone the omission of their 12

A1: Fully correct integration $\frac{1}{3}u^3\sin^{-1}u + \frac{1}{3}\sqrt{1-u^2} - \frac{1}{9}\left(\sqrt{1-u^2}\right)^3$ (may not have reverted back to being in terms of *u*). Condone any multiple and condone missing variables

M1: Applies the limits 0 and 1 to an expression of the form $Du^3\sin^{-1}u + E\sqrt{1-u^2} + F\left(\sqrt{1-u^2}\right)^3$ where *D*, *E* and $F \neq 0$

A1: Correct answer in the required form. $\frac{8}{3}$ does not need to be in simplest form. isw once a correct answer is seen.

Alt (b) – Integration by parts twice

M1: Attempts integration by parts in the correct direction to achieve an expression of the form $...t\sin^3 t \pm ...\int \sin^3 t \, \mathrm{d}t$ Condone the omission of their 12

M1: Proceeds to an integrable form by attempting integration by parts again achieving the form $...t\sin^3 t \pm ...\cos t\sin^2 t \pm ...\int \sin t\cos^2 t \, \mathrm{d}t$ Condone the omission of their 12

A1: $(12)\left(\frac{t}{3}\sin^3 t + \frac{1}{3}\cos t\sin^2 t + \frac{2}{9}\cos^3 t\right)$ o.e. Condone any multiple and condone missing variables

M1: Applies the limits 0 and $\frac{\pi}{2}$ to an expression of the form $pt\sin^3 t + q\sin^2 t\cos t + r\cos^3 t$ where p, q and $r \neq 0$

A1: As above in main scheme notes